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THE HEAT ENGINE, REFRIGERATOR AND HEAT PUMPS.

A thermal (heat) reservoir is that part of environment which can exchange heat energy with a system. It has sufficient large heat capacity and it temperature is not affected by the quantity of heat transferred to or from it. A thermal reservoir is thus characterised by its temperature which remains constant. The changes that do takes place in the thermal reservoir as heat enters or leave are so slow and so small that processes within it are quasi-static. The reservoir which is at high temperature and supplies heat is known as heat source (a). Examples are boiler furnace, a combustion chamber and a nuclear reactor etc. The reservoir which is at low temperature and to which heat is transferred is called the heat sink (b). Atmospheric air, ocean and river etc. constitute the heat sink.

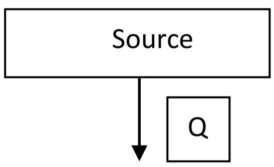


Fig. 1a Heat source

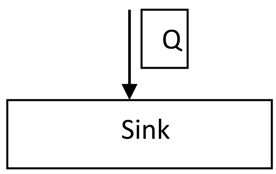


Fig 1b Heat sink.

A heat engine is a thermodynamic device used for continuous production of work from heat when operating in a cyclic process. Both heat and work interactions takes place across the boundary of the cylindrically operating device. A heat engine is characterised by following features:

- Reception of heat Q_1 from ahigh temperature source at T_1.
- Partial conversion of heat received to mechanical work W.
- Rejection of the remaining heat Q_2 to a low temperature sunk at temperature T_2.

- Cyclic/ continuous operation and
- Working substance flowing through the engine.

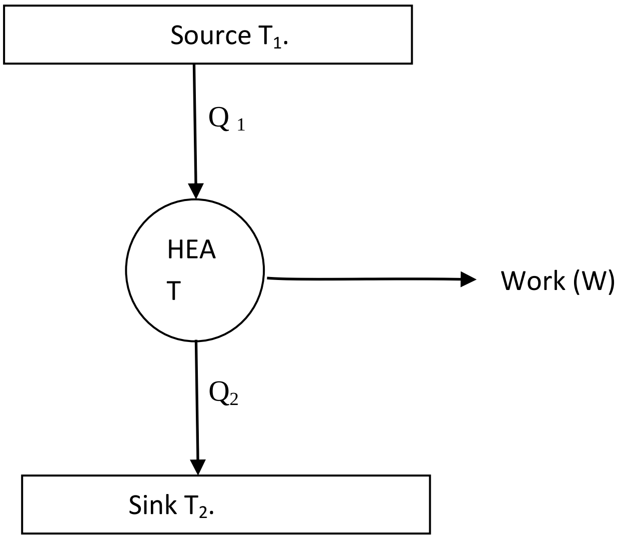


Fig. 2.Energy interaction in a heat

The performance of any machine is expressed as the ratio of 'what we want to what we have to pay for'. In the context of an engine, work is obtained at the expense of heat input. Accordingly, the performance of a heat engine is given by network output to the entire amount of heat supplied to the working medium, and this ratio is called thermal efficiency, η_{th} (Thermal efficiency is a measure of the degree of useful utilisation of heat received in a heat energy).

$$\eta_{th} = \frac{\text{net work output}}{\text{total heat supplied}}$$

Application of the principle of energy conservation (First law) to the heat engine, which undergo a cycle gives : $W = Q_1 - Q_2$

$$\therefore \eta_{th} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \quad ---- \quad (1)$$

Obviously, thermal efficiency of a heat engine operating between two thermal reservoirs is always less than unity. To increase the thermal efficiency, it is necessary to reduce Q_2 (heat rejected) with Q_1 (heat supplied) remaining constant. Thermal efficiency could be equal to unity if $Q_1 \rightarrow \infty$ and $Q_2 = 0$ which, however, cannot be realised in practice.

Refrigerators and heat pumps are reversed heat engines. The adjective 'revered' means operating backwards. The direction of heat and work interactions are opposite to that of a heat engine, i.e, work input and heat output. These machines (refrigerators and heat pumps) are used to remove heat from a body at low temperature level and then transfer this heat to another body at high potential of temperature. When the main purpose of the machine is to remove heat from the cooled space, it is called a refrigerator. A refrigerator operates between the temperature of surroundings and a temperature below that of the surroundings. Refrigerators are essentially used to preserve food items and drugs at low temperature.

The term heat pump is applied to a machine whose objective is to heat a medium which may already be warmer than its surroundings. A heat pump thus operates between the temperature of the surroundings and a temperature above that of the surroundings. Heat pumps are generally used to keep the rooms warm in winter. The transfer of heat against a reversed temperature gradient in a refrigerator and heat pump is accomplished by supplying energy to the machine. A schematic representation of heat pump and refrigerator is shown below.

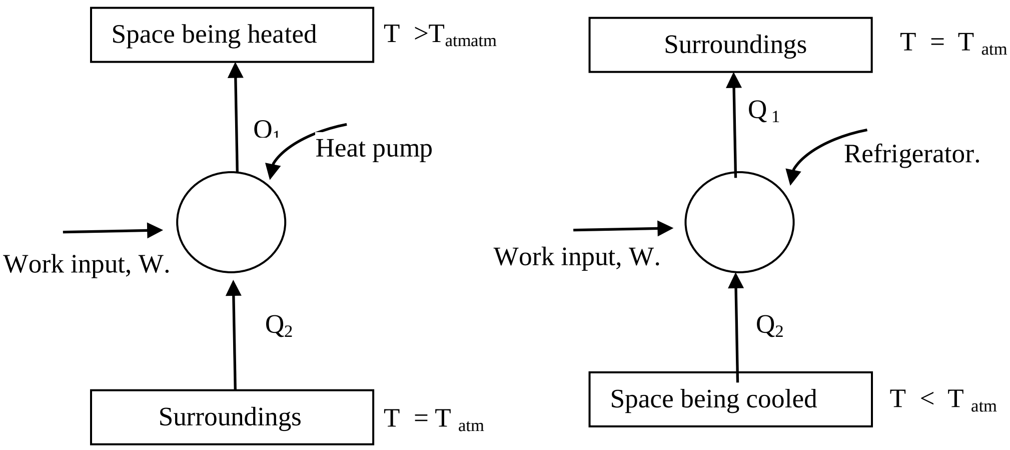


Fig. 3a. Fig. 3b

.Fig.3.Functional difference between a heat pump and a refrigerator.

The performance of a refrigerator and heat pump is expressed in term of coefficient of performance (COP) which represent the ratio of desired effect to work input.

$$\text{COP} = \frac{\text{desired effect}}{\text{work input}}$$

I a refrigerator, the desired effect is the amount of heat Q2 extracted from the space being cooled, i.e. the space at low temperature,

$$(\text{COP})_{\text{ref}} = \frac{\text{heat extracted at low temperature}}{\text{work input}} = \frac{Q_2}{W}$$

From energy conversion principle: $W = Q_1 - Q_2$.

$$\therefore\ (\text{COP})_{\text{ref}} = \frac{Q_2}{Q_1 - Q_2} \quad \ldots\ldots\ldots$$

For most of the refrigerating machines, the values of COP lie between 3 and 4, and the COP are greatest when temperature differences are least. In a heat pump, the desired effect is the amount of heat Q_1 supplied to the space being heated,

$$(\text{COP})_{\text{heat pump}} = \frac{Q_1}{Q_1 - Q_2} = 1 + \frac{Q_2}{Q_1 - Q_2} = 1 + (\text{COP})_{\text{ref}} \quad \ldots\ldots..$$

Thus the COP of a machine operating as a heat pump is higher than the COP of the same machine when operating as a refrigerator by unity.

EX.1. A heat engine produces work equivalent to 80 kW with an efficiency of 40%. Determine the heat transfer rate to and from the working fluid.

Solution: Ref to fig 2.

Thermal efficiency, $\eta_{th} = \frac{workoutput}{heatsupplied} = \frac{W}{Q_1}$ ∴ Heat supplied to engine,

$Q_1 = \frac{W}{\eta_{th}} = \frac{80}{0.4} = 200$ kW

EX.2. A heat engine develop a 10 kW power when receiving heat at the rate of 2250 kJ/min. Evaluate the corresponding rate of heat rejection from the engine and its thermal efficiency.

Solution: Ref to fig 2.

Thermal efficiency, $\eta_{th} = \frac{workoutput}{heatsupplied} = \frac{W}{Q_1}$. Given: W = 10 kW = 10 kJ/s = 600 kJ/min. and

Q_1 = 2250 kJ/min. ∴$\eta_{th} = \frac{600}{2250}$ 0.267 *or* 26.7 %, from the principle of energy conservation: $Q_1 = W + Q_2$. ∴ Amount of heat rejected, $Q_2 = Q_1 - W = 2250 - 600 = 1650$ kJ/min.

EX. 3. A reversed heat engine absorbs 250 kJ of heat from a low temperature region and has a mechanical input of 100 kJ. What would be the heat transferred to the high temperature region ?.

Also, evaluate the coefficient of performance of the reversed engine when working as a refrigerator and as a heat pump.

Solution: Ref to fig 3

From the principle of energy conservation (first law): $Q_1 = W + Q_2$. ∴ Heat transferred to the high temperature region, $Q_1 = W + Q_2 = 100 + 250 = 350$ kJ.

$$(COP)_{ref} = \frac{Q_2}{W} = \frac{250}{100} = 2.5$$

$$(COP)_{heat\ pump} = \frac{Q_1}{W} = \frac{350}{100} = 3.5$$

EX. 4. A machine operating as a heat pump extracts heat from the surrounding atmosphere, is driven by a 6.5 kW motor and supplies 2×10^5 kJ/hr heat to a house needed for its heating in winter. Find coefficient of performance for the heat pump. How this COP will be affected if the objective of the same machine is to cool the house in summer requiring 2×10^5 kJ/hr of heat rejection? Comment on the result.

Solution: Ref. to fig 3.

Work input to heat pump, W = 7.5 kW = 7.5 kJ/s = 0.27×10^5 kJ/hr. From the principle energy conservation: $W = Q_1 - Q_2$. ∴ Heat extracted from the surrounding atmosphere, $Q_2 = Q_1 - W$

$$= 2 \times 10^5 - 0.27 \times 10^5 = 1.73 \times 10^5 \text{ kJ/hr}$$

$$(\text{COP})_{\text{heat pump}} = \frac{\text{heat supplied to room } Q_1}{\text{work input (W)}} = \frac{2 \text{ x } 10^5}{0.27 \text{ x } 10^5} = 7.407$$

$$(\text{COP})_{\text{ref}} = \frac{\text{heat extracted from room } Q_2}{\text{work input (W)}} = \frac{1.73 \text{ x } 10^5}{0.27 \text{ x } 10^5} = 6.407$$

Apparently the same machine has two values of its performance depending upon the task being accomplished.

CONSERVATION OF ENERGY.

The concept of energy and the hypothesis that it can be neither created nor destroyed where developed by the scientists in the early part of the nineteenth century, and became known as the principle of conservation of energy. The first law of thermodynamics is merely one statement of this general principle with particular reference to heat energy and mechanical energy (i.e Work). The study of thermodynamic shows that when a system is made to undergo a complete cycle then net work is done on or by the system. Consider a cycle in which net work is done by the system. Since energy cannot be created, this mechanical energy must have been supplied from some source of energy. Now the system has been returned to its initial state, therefore its intrinsic energy is unchanged, and hence the mechanical energy has not been provided by the system itself. The only other energy involved in the cycle is the heat which was supplied and rejected in the various processes. Hence, by the principle of the conservation of energy, the net work done by the system is equal to the net heat supplied to the system. The first law of thermodynamic can therefore be stated as follows:

" When a system undergoes a thermodynamic cycle then the net heat supplied to the system from its surroundings is equal to the net work done by the system on its surroundings". In symbols,

$$\sum dQ = \sum dW$$

Where $\sum$ represents the sum for a complete cycle.

In summary, the law of conservation of energy state that,

i. Energy can neither be created nor destroyed; it is always conserved. However it can change from one kind to another.
ii. Total energy of an isolated system, in all its forms remains constant.
iii. All the energy that goes into a system comes out in some other form. Energy does not vanish and has the ability to be converted into any other form of energy.
iv. No machine can produce energy without corresponding expenditure of energy. Energy cannot appear from nothing, nor can it convert into nothing.
v. The first law of thermodynamics cannot be proved mathematically. Its validity stems from the fact that neither it nor any of its consequences have been contradicted.

FIRST LAW FOR A CYCLIC PROCESS.

A process is cyclic if the initial and final states of the system executing the process are identical. A system represented by state point 1 undergoes a process 1 - a – 2, and comes back to initial state following the path 2 – b – 1. All properties of the system are restored, when the initial state is reached. During the execution of these processes:

i. Area 1 – a – 2 – 3 – 4 – 1 represent the work W_1 done by the system during expansion process 1- a – 2 ,
ii. Area 2 – 3 – 4 – 1 – b – 2 represent the work W_2 supplied to the system during compression process 2 – b – 1
iii. Area 1 – a – 2 – b – 1 represent the net work ($W_1 - W_2$) delivered by the system.

Since the system regains its initial state, there is no change in the energy stored by the system. For energy balance and in accordance with the law of conservation of energy, an equivalent amount of heat energy must have been received by the system. Accordingly the first law of thermodynamics for a cyclic process can be mathematically expressed as:

$W_1 - W_2 = Q_1 - Q_2$; $\oint dQ = \oint dW$ ………

i.e, cyclic integral of heat $\oint dQ$ is equal to cyclic integral of work $\oint dW$; both forms of energy been expressed in the same unit; or

$\oint (dQ - dW) = 0$ ………..

It follows from the above equation that 'if a system is taken through a cycle of processes so that it returns to the same state or condition from which it started, the sum of heat and work effects will be zero'. It is to be noted that though both dQ and dW are path functions, their difference (dQ- dW) is a point function as the integral $\oint (dQ - dW) = 0$ ie, is zero.

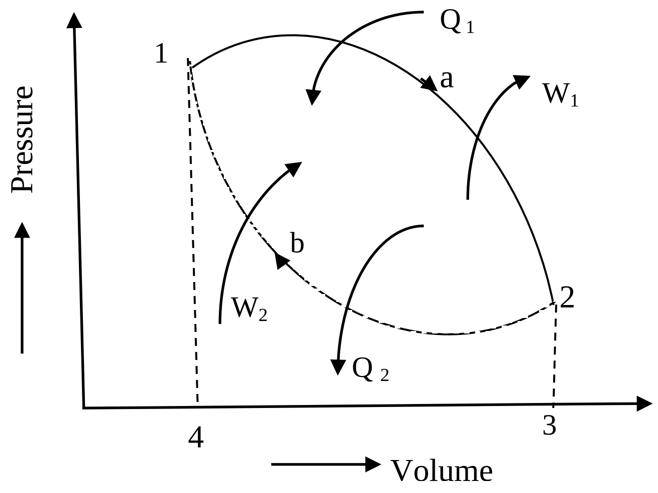


Fig. 4 Cyclic process

STORED ENERGY- A PROPERTY OF THE SYSTEM.

Consider a closed system which changes from state 1to state 2 by path A and retures back to original state 1 by any one of the following paths,

i. along path 2B1

ii. along path 2C1

iii along path 2D1

Invoking first law for the cyclic process 1- A – 2 – B – 1

$$\oint (dQ - dW) = 0$$

or $\int_{1,viaA}^{2} (\delta Q - \delta W) + \int_{2,viaB}^{1} (\delta Q - \delta W) = 0$ …….1

Likewise for the cycle 1- A – 2 – C – 1,

$\int_{1,via,A}^{2} (\delta Q - \delta W) + \int_{2,via,C}^{1} (\delta Q - \delta W) = 0$ …….2

and for the cycle 1 – A – 2 – D – 1

$\int_{1,via,A}^{2} (\delta Q - \delta W) + \int_{2,via,D}^{1} (\delta Q - \delta W) = 0$ …….3

Comparing equations 1,2,& 3; we may write:-

$$\int_{2,via,B}^{1} (\delta Q - \delta W) = \int_{2,via,C}^{1} (\delta Q - \delta W) = \int_{2,via,D}^{1} (\delta Q - \delta W)$$

Since B,C, and D represent arbitrary paths between the state 2 and state point 1, it can be concluded that the integral $\int_{2}^{1} (\delta Q - \delta W)$

i. remains the same irrespective of the path along which the system proceeds,

ii is solely dependent on the initial and the final state of the system; is a point function and hence a property.

The integral $\int_{2}^{1} (\delta Q - \delta W)$ is called energy of the system and is given the symbol E. Further energy is a property of the system; its differential is exact and is denoted by dE.

Thus for a process, $\delta Q - \delta W$ = dE; $\delta Q = \delta W +$ dE ……….4

The energy E is an extensive property. The specific energy, e = E / m, is an intensive property. In a reversible non-flow process, $\delta W = pdV$ and therefore,

$\delta Q = pdV + dE$ ………5

The energy E includes all forms of energy in the system, i.e,

E = U + KE + PE + CE + EE + MG +……. where energy U is associated with molecular motion and is stored in the molecules and atomic structure of the substance. In an ideal gas, there are no molecular forces of attraction and repulsion and theenergy U depends only on temperature, i.e U =f(T). Energy of this kind is called internal energy, intrinsic energy or microscopic energy. For a closed system, there is no flow of fluid into and out of the system and as such the KE of the mass of the system is not to be considered. There is no appreciable change in the PE of the system mass. Further, the process is quasi-static in nature and the system moves from one equilibrium state to another. Thus no chemical reaction or change in chemical composition is involved and hence no change in chemical energy of the system.

In the absence of KE, PE, CE, EE, & Magnetic effects, equations 4 & 5 can be rewritten as

$\delta Q = \delta W +$ dU; or $\delta Q = pdV + dU$ 6

Equation 6 is an analytical expression of the first law of thermodynamics and often referred to as the basic equation of thermodynamics. It shows that the heat received by a system is utilized to change its internal energy and to make the system perform external work. If the expansion of the system is without any transfer of heat ($\delta Q = 0$), the work will be performed at the cost of internal energy. Further, if the system receives heat without any change in its boundaries ($\delta W = 0$), the entire heat added will go in for increasing the internal energy of the system.

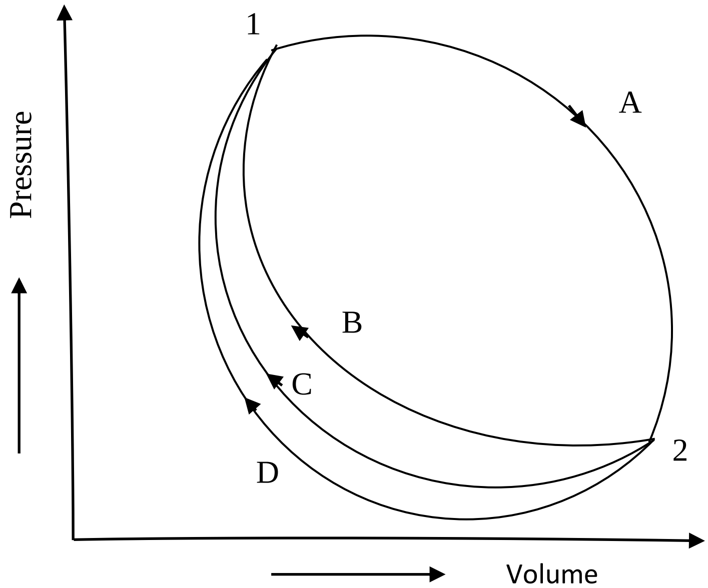


Fig 5.Energy – a property of the system.

Ex. 1.

A system undergoes a cycle composed of four processes. The heat transfers in each process are: 400kJ, -365kJ, -200kJ, and 250kJ. The respective work transfers are: 140kJ, 0, -55kJ, and 0. Is the data consistent with first lawof thermodynamics?

Solution: Net heat interaction,

$$\oint \delta Q = 400 - 365 - 200 + 250 = 85kJ$$

Network transfer, $\oint \delta W = 140 + 0 - 55 + 0 = 85kJ$

Since $\oint \delta Q = \delta W$, the given data conforms to the first law of thermodynamics.

Ex 2.

The working fluid in an engine executes a cyclic process, and two work interactions are involved: 10kJ to the working fluid and 30 kJ from the working fluid. The working cyclic also involves three heat transfers two of which are 75 kJ to the working fluid and 40 kJ from the working fluid. Determine the magnitude and direction of the third heat interaction.

Solution: $\oint \delta W = -10 + 30 = 20kJ$

$\oint \delta Q = 75 - 40 + x$

Where x is the third heat interaction. From the first law of thermodynamics,

$\oint \delta Q = \delta W$ for a cyclic process.

∴ 75 – 40 + x = 20

∴ x = -15 kJ, the negative sign indicates that heat flows out of the system.

Ex 3.

Consider a steam turbine power plant system where the work and heat interactions across the system boundaries are:

i. Power available at the turbine shaft is 600 kW and the feed pump requires 3 kW of work to pump the condensate back into the boiler.

ii. Steam generation in the boiler requires 2500 kJ/kg of heat, and in the condenser 1800kJ/kg of heat is rejected to cooling water.

Make calculations for the steam flow rate in kg/hr round the cycle.

Solution:

$\oint \delta W = 600 - 3 = 597 kW = 597 \times 3600 kJ/hr$

If m is the steam flow rate in kg/hr, then

$\oint \delta Q$ = m(2500 – 1800) = 700m kJ/hr.

From the first law of thermodynamics,

$\oint \delta Q = \delta W;$ 700m = 597 x 3600

∴ m = $\frac{597 \, x \, 3600}{700} = 3070 kg/hr.$

Ex 4.

In a general compression process, 2 kJ of mechanical work is applied to 5 kg of working substance, and 800 J of heat is rejected to the cooling jacket. Calculate the change in specific internal energy.

Solution: From non-flow energy equation, $\delta Q = \delta W + dU$

$\therefore dU = \delta Q - \delta W$

= - 800 – (- 2000) = 1200 J.

The heat transfer has been taken negative it flows of system. Further work is also negative as this has been supplied to the system.

du = change in internal energy per unit mass,

$\frac{1}{5}\ x\ 1200 = 240\ J/\ kg.$

Ex.5

A domestic refrigerator is loaded with fresh food and the door closed. During a certain period, the machine consumes 1.2kWh of electrical energy in cooling the food and the internal energy of the system decreases by 4500kJ. Find the magnitude and sign of net heat transfer for the system. The refrigerator and its contents may be considered as the system.

Solution: Work supplied to refrigerator, $\delta W = 1.25kWh = 1.25 \times 3600 = 4500kJ$

This work interaction is negative as work has been supplied to the system.

Change in internal energy, dU = -4500 kJ (since decreases)

From non-flow energy equation,

$\delta Q = \delta W + dU$, dQ = - 4500 + (- 4500) = - 9000 kJ. Since this is negative, heat flows out of the system.

Ex.6

A storage battery, having a terminal potential of 12 volts, draws a current of 8 amperes for 2.5 hrs. If the stored energy of the battery decreases by 1250 kJ, evaluate the heat interaction across the boundary enveloping the storage battery.

Solution: Work done is 2.5hrs. = $\frac{V}{\tau}$ = $\frac{12 \times 8 \times 2.5}{1000}$ kWh

= $\frac{12 \times 8 \times 2.5}{1000} \times 3600$ = 864 kJ

This is a positive work as the current flows out of the battery. Further, the battery forms a closed system, and then from the non-flow energy equation,

$\delta Q = \delta W + dU$ = 864 + (-1250) = 386 kJ.

ENERGY CONVERSION.

Consider water available in the natural or artificial high level water reservoir. When the water falls, its potential energy gets transferred to kinetic energy. The high velocity jet impinges on the blades of a turbine and set it on a rotary motion. The mechanical energy made available at the turbine shaft is utilized to run an electric generator which is coupled to the turbine shaft. The electrical power thus generated, if shorted through a resistance coil produces heat. The different species of energy, though qualitatively different, are simply diverse manipulations of a single fundamental potency namely energy and disappearance of one form gives way to another form.

Some other notable examples of energy transformations are:

- The heat of fuel input to an internal combustion engine can be accounted for as output in the form of mechanical energy at the shaft and heat loss to cooling medium and surroundings etc.
- When a running automotive vehicle is brought to rest by applying brakes, the kinetic energy of the vehicle is converted into heat energy through friction.
- There occurs conversion of potential energy into kinetic energy when a fluid flows through a pipeline.
- During flow of current through a resistance, the electrical energy is converted into heat.

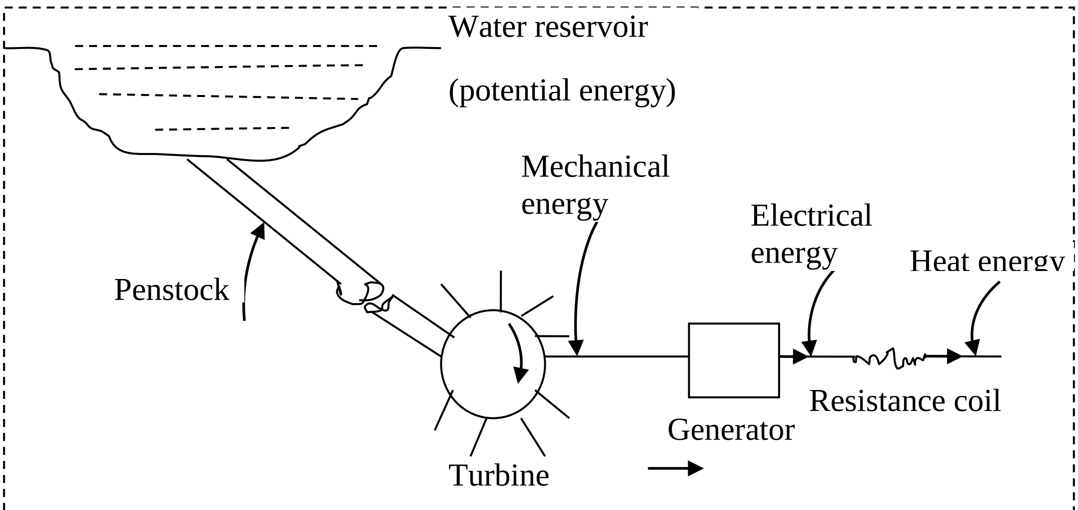


Fig.5 Energy conversion

LIMITATIONS OF FIRST LAW.

First law of thermodynamics deals with conservation and conversion of energy. It stipulates that when a thermodynamic process is carried out, energy is neither gained nor lost. Energy only transforms from one form into another and the energy balance is maintained. The law, however, fails to state the condition under which energy conversions are possible. The law presumes that any change of a thermodynamic state can take place in either direction. However, this is not true; particularly in the inter- conversion of heat and work. Processes proceed spontaneously in certain directions but not in opposite directions, even though the reversal of processes does not violate the first law.

For example:

i. Consider a running automobile vehicle stopped by applying brakes. The process changes the kinetic energy of the vehicle into heat and the brakes get heated up. The increase in the internal energy of the brakes is in accordance with the first law. The stopping of the vehicle by friction, however, constitutes an irreversible process. Cooling of brakes to their initial state never puts the vehicle back into motion. Heat in the brakes cannot be converted back into mechanical work even though that act would not violate the principle of energy conversion.

ii. The temperature of liquid contained in a vessel rises when it is churned by paddle work. However, the paddle work cannot be restored on cooling the liquid to its original state.

iii. Consider two bodies A and B at different temperatures t_A and t_B ($t_A > t_B$) in thermal contact into an insulated box. Energy in the form of heat will flow from body A to body B because of temperature difference. This energy flow continues till both the bodies attain the same temperature level at some intermediate value between these original temperatures. In conformity with the first law, heat lost by body A equals the heat gained by body B. First law would be equally satisfied if the reverse process were to occur i.e., the energy in the same amount get transformed from body B to body A so that both bodies regain their initial temperatures. This, however, is never seen to happen.

iv. When a block slides down a rough plane, it becomes warmer. However, the reverse process where the block slides up the plane and become cooler is not true even though the first law will still hold good.

v. Work converts easily and completely into heat. However, there does exist a maximum limit up to which conversion of heat is possible in a heat engine. As a source of energy, work is superior to heat, and a complete conversion of low grade energy (heat) into high grade energy (work) is impossible.

vi. Electric current flowing through a resistor produces heat. Electric current once dissipated as heat cannot be converted back into electricity.

vii. Water flows from a higher level to a lower level, and the reverse is not automatically possible. A mechanical energy from an external source would be required to pump the water back from the lower level to higher level.

viii. Fuel (Coal, diesel, petrol) burns with air to form the products of combustion. Fuel once burnt cannot be restored back to its original form.

ix. When hydrogen and oxygen are kept in an isolated system, they produce water on chemical initiation. However, the water left in the isolated system never dissociates into hydrogen and oxygen.

x. Persons always grow old………..

The examples given above indicated that:

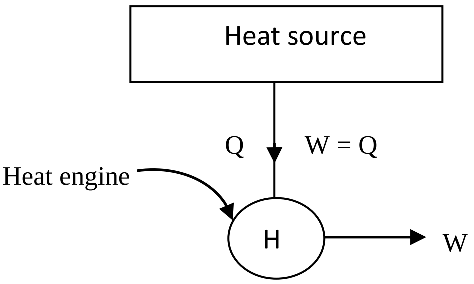
- First law fixes the exchange rate between heat and work, and places no restrictions on the direction of change.
- Processes proceed spontaneously in certain directions, but the reverse is not automatically attainable even though the reversal of the processes does not violate the first law.
- First law provides a necessary but not a sufficient condition for a process to occur.
- There does exist some directional law which would tell whether a particular process occurs or not. Answer is provided by the second law of thermodynamics.

THE SECOND LAW OF THERMODYNAMICS.

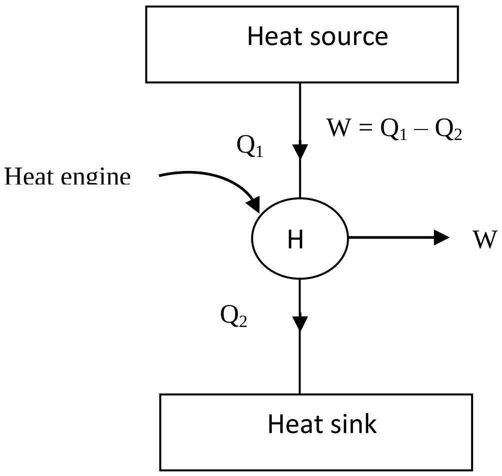
KELVIN-PLANCK STATEMENT.

"It is impossible to construct an engine that operates in a cycle and produces no effect other than work output and exchange of heat with a single reservoir"

The statement implies that no heat engine can be developed that receives a certain amount of heat from a high temperature source and converts that into an equivalent amount of work. i.e W = Q. The thermal efficiency of such an engine $\eta = W/Q = 1$ or 100%. Fig 1 (a) represents the schematic arrangements of a heat engine that exchanges heat with a single heat source and is 100 % efficient. Such a system satisfies the principle of energy conservation (1^{st} law) but violates the Kelvin statement of second law. Obviously Kelvin-Planck statement tells that no heat engine can have thermal efficiency equal to 100 %.



(a) Impossible



(b) Possible

Fig. 1. Schematic representation of a heat engine

The only alternative for continuous power output from a heat engine is that a portion of the heat received must be rejected to a heat reservoir at low temperature (heat sink). This engine receives Q1 units of heat, rejects Q2 units of heat and converts (Q1 – Q2) units of heat into work per cycle. All possible heat engines conform to this representation (Fig 1 (b)).

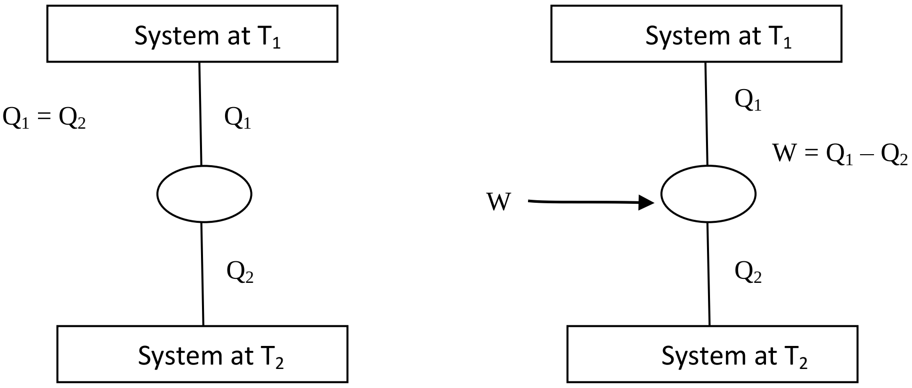
CLAUSIUS STATEMENT.

"It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a system at low temperature to another system at high temperature".

The statement implies that heat cannot flow of itself from a system at low temperature to a system at high temperature. The schematic arrangement that is prohibited by Clausius statement is shown in fig 2 (a). The coefficient of performance of such an arrangement equals:

COP = Q/W = Q/0 = ∞

Obviously the Clausius statement tells that COP of a heat pump/refrigerator cannot be equal to infinity. The only alternative for the transfer of heat from low temperature to high temperature level is that some external work must be supplied to the machine as shown in fig 2 (b).



(a) Impossible (b) possible

Fig. 2: schematic representation of a heat pump/refrigerator

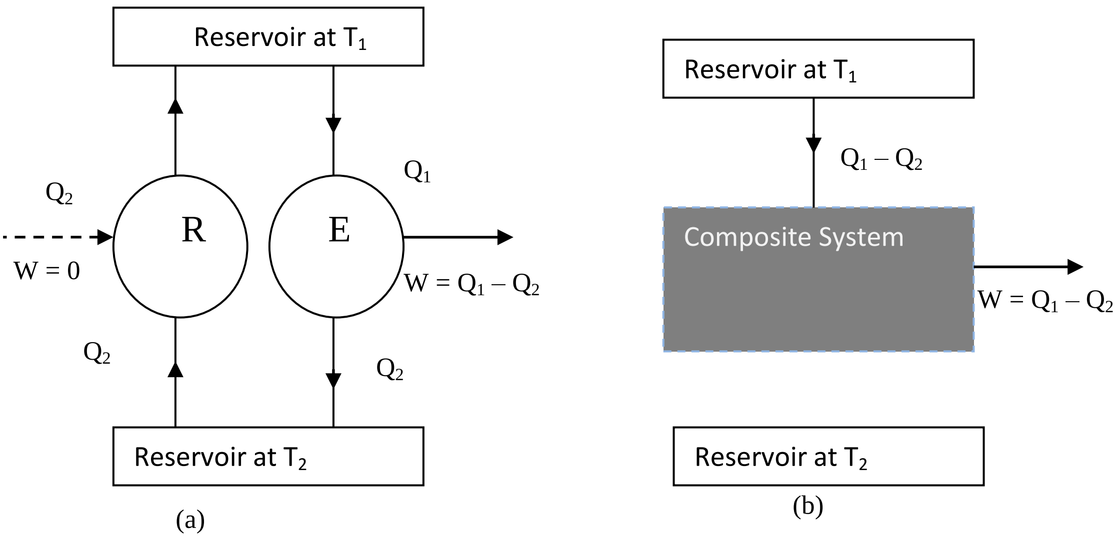
Whereas the Kelvin- Planck statement is applied to heat engines, the Clausius statement concerns heat pumps and refrigerators. Both the Kelvin-Planck and Clausius statements are negative statements, they have no mathematical proof. The law is based on experimental observations, and to-date no observations have been made that contradicts the law and this aspect is taken as sufficient evidence of its validity. The Kelvin – Planck and Clausius statements, though worded differently, are interlinked and are complementary to each other. It is impossible to have a device satisfying one statement and violating the other. Any device that violates Clausius statement lead to violation of Kelvin- Planck statement and vice- versa.

VIOLATION OF CLAUSIUS STATEMENT LEADING TO VIOLATION OF KELVIN – PLANCK STATEMENT.

Fig.3 (a) depicts a refrigerator R that operates in a cycle and transfers Q_2 units of heat from a low temperature reservoir at T_2 to a high temperature reservoir at T1 without any work input from an external agency (surroundings). This is in violation of Clausius statement. Indicated along-with is a heat engine E that too operates in a cycle. This engine takes Q1 units of heat from the high temperature reservoir, delivers ($Q_1 – Q_2$) units of work to the surroundings and rejects the remaining Q_2 units of heat to the low temperature reservoir. The engine operates in conformity with the Kelvin – Planck statement.

Fig 3 (b) illustrates the heat and work interactions for refrigerator and heat engine when coupled together. This composite system constitutes a device that receives ($Q_1 – Q_2$) units of heat from the high temperature reservoir and converts it completely into an equivalent amount of work W = ($Q_1 - Q_2$) without rejecting any heat to the low temperature reservoir. This operation of the composite system is in violation of Kelvin – Planck statement.

Fig. 3: Violation of Clausius statement lead to violation of Kelvin – Planck statement



Thus, violation of Clausius statement leads to violation of Kelvin – Planck statement also.

VIOLATION OF KELVIN – PLANCK STATEMENT LEADING TO VIOLATION OF CLAUSIUS STATEMENT.

Fig.4 (a) depicts the engine E which operates from a single heat reservoir at temperature T_1. It receives Q_1 units of heat from this reservoir and converts it completely into an equivalent amount of work W = Q_1 without rejecting any heat to the low temperature reservoir at T_2. This is in violation with the Kelvin – Planck statement. Indicated along-with it is a refrigerator R which extracts Q_2 units of heat from the low temperature reservoir, is supplied with Q_1 units of work from an external agency (surroundings) and supplies (Q_1+ Q_2) units of heat to the high temperature reservoir. The refrigerator operates in conformity with the Clausius statement.

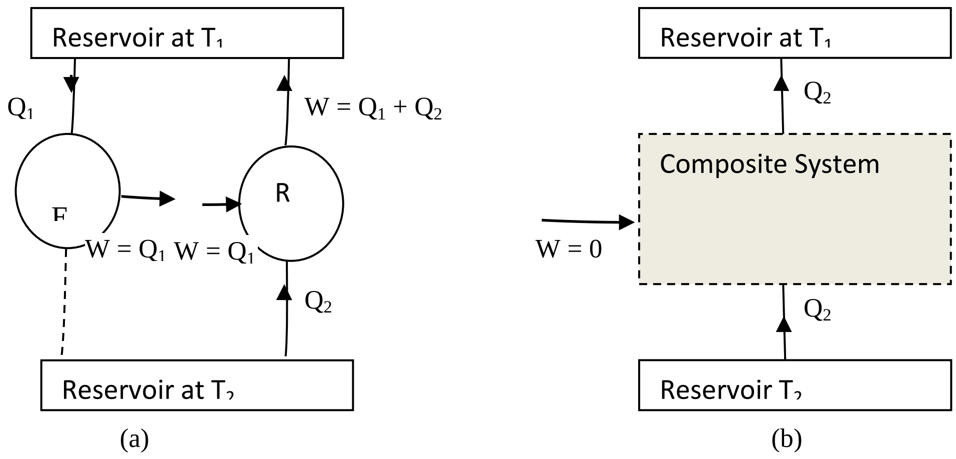


Fig. 4: Violation of Kelvin – Planck statement leads to Clausius statement.

Fig. 4 (b) illustrates the work and heat interactions for the refrigerator and heat engine when coupled together. The output of engine is utilized to drive the refrigerator. This composite system constitutes a device which transfers heat from the low temperature reservoir to the high temperature reservoir without any work input from an external agency (surroundings). This is in violation of the Clausius statement. Thus, violation of Kelvin – Planck statement lead to violation of Clausius statement also.

REVERSIBLE AND IRREVERSIBLE PROCESSES.

A thermodynamic process is reversible if both the system and surroundings can be restored to their respective initial states by reversing the direction of the process. A reversible process is a quasi-static process, a process carried out infinitely slowly with infinitesimal gradient with the system passing through a continuous series of equilibrium states. The reversible processes can proceed in either direction without violating the second law of thermodynamics. A system going through a reversible process results in maximum efficiency. In other words, a reversible process yields the maximum work in engines and requires minimum work in devices such as refrigerators, pumps and compressors. All natural processes take place simultaneously at finite speeds through finite discontinuities between the system and its environment and as such are irreversible. Processes are rendered irreversible as a result of degradation of energy by such factors as friction, turbulence, diffusion, inelasticity and electrical resistance etc. These elements can be reduced but can not be completely eliminated.

CARNOT CYCLE AND CARNOT HEAT ENGINE.

A Carnot cycle is a hypothetical cycle consisting of four distinct processes: two reversible isothermal processes and two adiabatic reversible processes. The cycle was proposed in 1824 by a young Franch engineer Sadi Carnot. The essential elements needed for making an analysis of this cycle are:

- A working substance which is assumed to be a perfect gas.
- Two heat reservoirs; the hot reservoir (heat source) at T_1 and the cold reservoir (heat sink) at temperature T_2.

- Piston – cylinder arrangement for getting the work out of the working substance. The piston and cylinder walls (excluding the cylinder head) are taken as perfect heat insulators. The cylinder head is imagined to provide alternatively diathermic cover (perfect heat conductor) and an adiabatic cover (perfect heat insulator).

The cylinder – piston arrangement is shown in fig. 5. There is no friction to the movement of piston inside the cylinder.

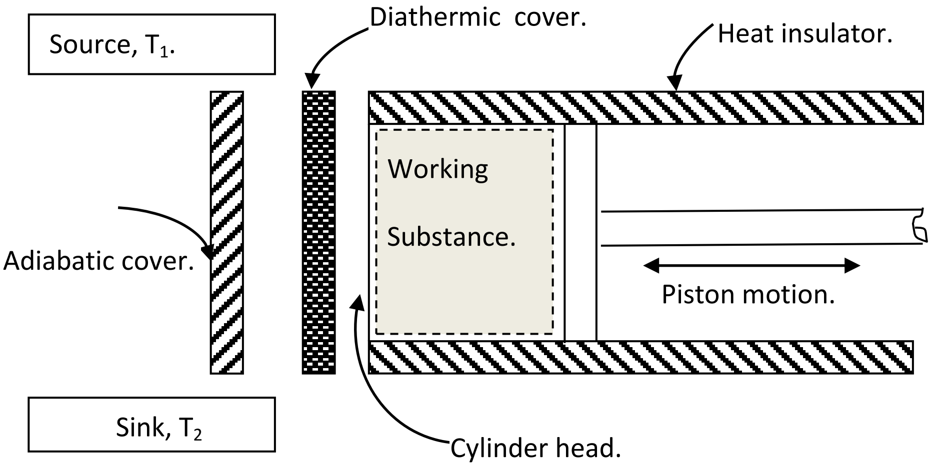


Fig. 5: Essential elements for a Carnot cycle.

The sequence of operation for the different processes constituting a Carnot cycle is:

xpansion (a- b): The heat is supplied to the working fluid at constant temperature T_1. This is achieved by bringing the heat source in good thermal contact with the cylinder head through diathermic cover. The gas expands isothermally from state point $a(p_a, V_a)$ to state point $b(p_b, V_b)$. The heat supplied equals the work done which is represented by area under the curve

a – b on pressure – volume plot and is given by,

$$Q_1 = W_{a-b} = p_a V_a log_e \frac{V_b}{V_a}$$

$$= mRT_1 log_e \frac{V_b}{V_a}$$

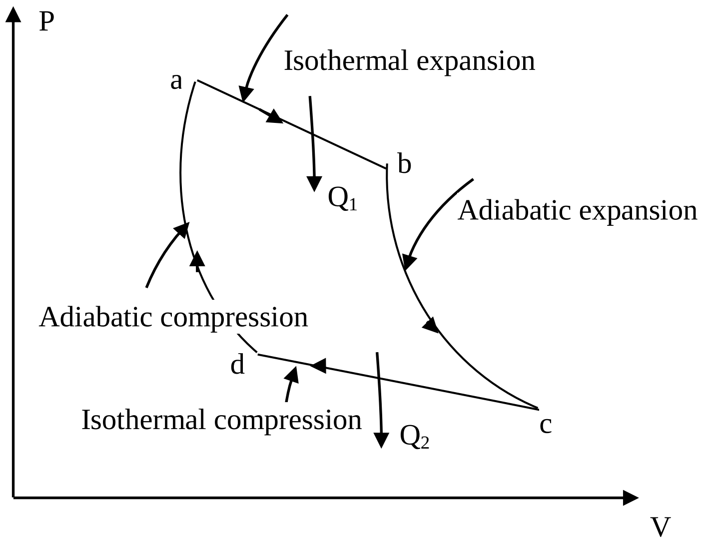


Fig. 6: Carnot cycle on P – V plot.

Adiabatic expansion (b – c): At the end of isothermal expansion (state point b), the heat source is replaced by adiabatic cover. The expansion continued adiabatically and reversibly up-to state point $c(p_c, V_c)$. Work is done by the working fluid at the expense of internal energy and its temperature falls to T_2 at state point c.

Isothermal compression (c – d): After state c, the piston starts moving inwards and the working fluid is compressed isothermally at temperature T_2. The constant temperature T2 is maintained by moving the adiabatic cover and bringing the heat sink in contact with the the cylinder head. The compression continues up-to state point d and this position of the piston is so chosen that it lies on the path of reversible adiabatic drawn from state point a. The working fluid loses heat to the sink and its amount equals the work done on the working fluid. This work is represented by area under curve c – d and its amount is given by,

$$Q_2 = W_{c-d} = p_c V_c log_e \frac{V_c}{V_d}$$

$$= .mRT_2 log_e \frac{V_c}{V_d}$$

Adiabatic compression (d – a):At the end of isothermal compression(state point d), the heat sink is removed and is replaced by adiabatic cover. The compression now proceeds adiabatically and reversibly till the working fluid returns back to its initial state point a. Work is done on the working fluid, the internal energy increases and temperature is raised to T_1. Since all the processes that constitute a Carnot cycle are reversible, the Carnot cycle is referred to as reversible cycle. Further, a cyclic heat engine working on the Carnot cycle is called Carnot engine and its thermal efficiency is given by;

$$\eta = \frac{net\ work\ output}{heat\ input} = \frac{W_{net}}{Q_1}$$

There are no heat interactions along the reversible adiabatic processes b – c and d – a and application of the first law of thermodynamics for the complete cycle gives;

$\delta W = \delta Q$

or$\boldsymbol{W_{net} = Q_1 - Q_2 = mRT_1 log_e \frac{V_b}{V_a} - mRT_2 log_e \frac{V_c}{V_d}}$

$$\therefore \eta = \frac{\boldsymbol{mRT_1 log_e \frac{V_b}{V_a} - mRT_2 log_e \frac{V_c}{V_d}}}{\boldsymbol{mRT_1 log_e \frac{V_b}{V_a}}}$$

$$= 1 - \frac{T_2}{T_1} \times \frac{\boldsymbol{log_e \frac{V_c}{V_d}}}{\boldsymbol{log_e \frac{V_b}{V_a}}} \ldots\ldots\ldots\ldots 1$$

For the adiabatic expansion process b – c and d – a,

$$\frac{T_b}{T_c} = \left(\frac{V_c}{V_b}\right)^{\gamma-1} and \frac{T_a}{T_d} = \left(\frac{V_d}{V_a}\right)^{\gamma-1}$$

Since $T_b = T_a = T_{1'}\ and\ T_c = T_d = T_{2'}$ the above expression give

$\frac{T_1}{T_2} = \left(\frac{V_c}{V_b}\right)^{\gamma-1} = \left(\frac{V_d}{V_a}\right)^{\gamma-1}$ or$\frac{V_c}{V_b} = \frac{V_d}{V_a}$ or $\frac{V_c}{V_d} = \frac{V_b}{V_a}$

Substituting the above relation in relation (1), we get,

$$\eta = 1 - \frac{T_2}{T_1} = \frac{T_1 - T_2}{T_1} \quad \ldots\ldots..2$$

Following conclusions can be made with respect to efficiency of a Carnot engine:

1. The efficiency is independent of the working fluid and depends upon the temperatures of source and sink.
2. If $T_2 = 0$, the engine will have an efficiency of 100 %. However, that means absence of heat sink which is violation of Kelvin – Planck statement of the second law.
3. The efficiency is directly proportional to temperature difference (T_1 –T_2) between the source and sink.
4. The higher the temperature difference between source and sink, the higher will be the efficiency obtained.
5. The efficiency increases with an increase in temperature of source and a decrease in temperature of sink.

6. If $T_1 = T_2$, no work will be done and efficiency will be zero.

Metallurgical considerations and the high cost of temperature resisting materials limit the higher temperature T_1. The lower temperature T2 is limited by atmospheric or sink conditions.

CARNOT CYCLE IS IMPRACTICABLE.

Carnot cycle gives the maximum possible thermal efficiency which can be obtained for the given temperature limits. The Carnot engine, however, is a hypothetical device and it is not possible to devise it due to the following reasons:

a. All the four processes have to be reversible. This necessitates that the working fluid must have no internal friction between the fluid particles and no mechanical friction between the piston and cylinder walls.

b. The heat absorption and rejection have to take place with infinitesimal temperature differences. Accordingly, the rate of energy transfer will be very low and the engine will deliver only infinitesimal power.

c. For attaining isothermal operation, the piston movement is required to be very slow. However, the piston must move fast for the adiabatic process. A variation in the speed of the piston during different processes of a cycle is rather impossible.

d. There is insignificant difference in the slopes of isothermal and adiabatic lines. Consequently the p – V plot is greatly extended both in the horizontal and vertical directions. The cylinder then involves greater pressures and volumes, and as such becomes bulky and heavy.

This hypothetical device, however, serves as a yardstick or standard of perfection against which the performance of any practical heat engine can be compared.

REVERSIBLE HEAT ENGINE (CARNOT HEAT PUMP).

Refrigerators and heat pumps are reversed heat engines. The adjective “reversed” means that the reversible processes constituting a heat engine are individually reversed and carried out in reversed order. When a reversible process is operated backwards all the energy transfers associated with the process get reversed in direction, but remain the same in magnitude.

Fig. 5, shows the p – V plot of a Carnot heat pump (i.e reversed Carnot heat engine). The sequence of operation is:

a – d: Isentropic (reversible adiabatic) expansion of working fluid in the clearance space of the cylinder. The temperature falls from T_1 to T_2.

d – c: Isothermal expansion during which heat Q_2 is absorbed at temperature T_2 from the space being cooled.

c – b: Isentropic compression of working fluid. The temperature rises from T_2 to T_1.

b – a: Isothermal compression of working fluid during which Q1 is rejected to a system at high temperature.

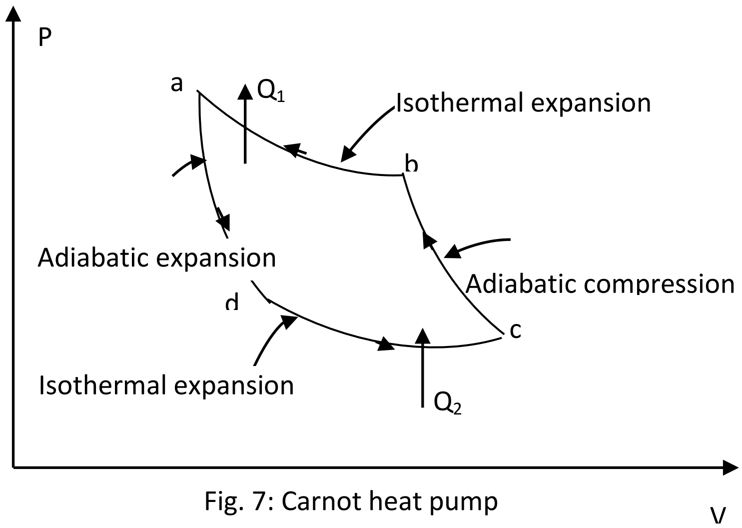


Fig. 7: Carnot heat pump

From the mathematical processes of equation (1) above,

$Q_1 = mRT_1 log_e \frac{V_b}{V_a}$; $Q_2 = mRT_2 log_e \frac{V_c}{V_d}$, also $\frac{V_c}{V_d} = \frac{V_b}{V_a}$,

Therefore, for a heat pump,

$$(COP)_{heat-pump} = \frac{Q_1}{Q_1 - Q_2} = \frac{\boldsymbol{mRT_1 log_e \frac{V_b}{V_a}}}{\boldsymbol{mRT_1 log_e \frac{V_b}{V_a} - mRT_2 log_e \frac{V_c}{V_d}}} = \frac{T_1}{T_1 - T_2} \text{ ….3}$$

For a refrigerator,

$$(COP)_{ref} = \frac{Q2}{Q_1 - Q_2} = \frac{\boldsymbol{mRT_2 log_e \frac{V_c}{V_d}}}{\boldsymbol{mRT_1 log_e \frac{V_b}{V_a} - mRT_2 log_e \frac{V_c}{V_d}}} = \frac{T_2}{T_1 - T_2} \quad \text{….. 4}$$

Although theoretically more efficient, Carnot heat pumps like the Carnot heat engines have not been used in practice. This is because the machine has to run at high speed during portions of the cycle when adiabatic processes are being performed and run very slowly during other portions when isothermal processes are being carried out. The variation of speed during a stroke is not practicable.

Ex. 1.

A heat engine working on Carnot cycle convert one-fifth of the heat input into work. When the temperature of the sink is reduced by 80^0C, the efficiency gets doubled. Make calculations for the temperature of source and sink.

Solution: The Carnot heat engine is a reversible heat engine and its efficiency is given by:

$\eta = \frac{T_1 - T_2}{T_1}$. In the first case: $\frac{T_1 - T_2}{T_1} = \frac{1}{5}$; i.e $5T_1 - 5T_2 = T_1$

$\therefore T_1 = 1.25T_2$.

In the second case

$\frac{T_1 - (T_2 - 80)}{T_1}$ = 2 x 1/5 = 2/5

$5T_1 - 5T_2 + 400 = 2T_1$.; $3T_1 + 400 = 5T_2$.

Substituting, T1 1.25T2 as obtained above,

$3(1.25T_2) + 400 = 5T_2$.; T2 = 320 K

∴ Temperature of sink, T2 = 320 K, and temperature of source, T1 = 1.25 x 320 = 400 K.

Ex. 2:

A reversible heat engine delivers 0.6 kW power and rejects heat energy to a reservoir at 300 K at the rate of 24 kJ/min. Make a calculations for the engine efficiency and the temperature of the thermal reservoir supplying heat to the engine.

Solution: Work output, W = 0.6 kW = 0.6 kJ/s.

Heat rejected, Q_2 = 24kJ/min = 24kJ/60 = 0.4 kJ/s,

From the principle of energy conservation (1^{st} law): $Q_1 = W + Q_2$.

∴ Heat supplied to the engine, Q_1 = 0.6 + 0.4 = 1.0 kJ/s,

Thermal efficiency, $\eta_{th} = \frac{Work\ output}{heat\ supplied} = \frac{0.6}{1.0}$ = 0.6 = 60 %.

For a reversible heat engine, thermal efficiency is also given by,

$\eta_{th} = \frac{T_1 - T_2}{T_1}$; $0.6 = \frac{T_1 - 300}{T_1}$, 0.6T1 = T1 – 300 or T1 = 300/0.4 =750 K.

Ex. 3:

An engine mounted on a ship has a thermal efficiency 80 % of that of the corresponding Carnot cycle. The engine receives heat from the sea at 300 K and rejects heat to the atmosphere at 280 K. The work output from the engine is dissipated through an agitator to heat 500 kg of sea water to 355 K. What quantity of heat must be extracted from the sea water to provide the required heating effect?

Take specific heat of sea water C_p = 4.186 kJ/kg K.

Solution: Efficiency of Carnot cycle,

$$\eta_{th} = \frac{T_1 - T_2}{T_1} = \frac{300-280}{300} = \frac{20}{300}$$

Thermal efficiency of the engine = 0.8 x $\frac{20}{300}$ = 0.0533

Energy (work) needed for heating the water,

$mc_p\Delta T$ = 500 x 4.186 (355 – 300) = 115115 kJ.

In terms of heat and work interactions;

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{W}{Q_1}$$

∴ Heat to be extracted from sea water,

$Q_1 = \frac{W}{\eta} = \frac{115115}{0.0533}$ = 2159756 kJ.

Ex. 4

A Carnot refrigerator operates between temperature -10^{0}C and 30^{0}C.Determine its coefficient of performance. Subsequently it is desired to change the temperatures so as to make the COP exactly equal to 7.0. The amount of increase in the higher temperature is equal to amount of decrease in the lower temperature. Work out the new temperatures.

Solution: T_1 = 30^{0}C = 303 K and T_2 = -10^{0}C = 263 K

COP = $\frac{T_2}{T_1 - T_2}$ = $\frac{263}{303-263}$ = 6.575

Let the change in temperature be x, then,

COP = 7 = $\frac{263-x}{(303-x)-(263-x)}$ = $\frac{263-x}{40+2x}$ or 280 + 14x = 263 – x

x = - 1.133 deg.

Hence, T_1 = 303 -1.133 = 301.867 K and T_2 = 263 – (-1.133) = 264.133 K.

Ex. 5

Which is more effective way to increase the efficiency of a Carnot heat engine?:

i. To increase the source temperature T_1 while the sink temperature T_2 is held constant.

ii. To decrease the sink temperature by the same amount while the source temperature is held constant.

How this result would be affected in case of a Carnot heat pump?

Solution: Let η_1 be the efficiency when the source temperature is increased by dT while the sink temperature is held constant, and η_2 be the efficiency when the sink temperature is decreased by dT while the source temperature is held constant. Then,

$$\eta_1 = \frac{T_1 - T_2}{T_1} = \frac{(T_1 + dT) - T_2}{T_1 + dT} \quad \ldots\ldots\ldots\ldots.1$$

$$\eta_2 = \frac{T_1 - T_2}{T_1} = \frac{T_1 - (T_2 - dT)}{T_1} = \frac{(T_1 + dT) - T_2}{T_1} \quad \ldots\ldots\ldots\ldots2$$

It is to be noted that,

i. Numerators for the expressions 1 and 2 are the same.

ii. Denominator (T_1 + dT) in expression 1 is more than the denominator T_1 in expression.

$\therefore \eta_2 > \eta_1$; hence, a decrease in the sink temperature while the source temperature is held constant is more effective in increasing the efficiency of a Carnot heat engine.

(b) Let $(COP)_1$ be the COP of the heat pump when the temperature of the high temperature reservoir is increased by dT while the low temperature reservoir is held at constant temperature, and (COP)2 be the COP if the temperature of the low temperature reservoir is decreased by dT while the high temperature reservoir is held at constant temperature. Then,

$$(COP)_1 = \frac{T_1}{T_1 - T_2} = \frac{T_1 + dT}{(T_1 + dT) - T_2} \quad \ldots\ldots\ldots\ldots\ldots.3$$

$$(COP)_2 = \frac{T_1}{T_1 - T_2} = \frac{T_1}{T_1 - (T_2 - dT)} = \frac{T_1 + dT}{(T_1 + dT) - T_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots4$$

An examination of expression 3 and 4 shows that $(COP)_1 > (COP)_2$. That is, an increase in the high temperature reservoir while the temperature of low temperature reservoir is held constant is more effective in increasing the COP of a heat pump. Note: refer to the class lecture notes on refrigerator systems and heat pumps.

The perfect gas.

The characteristic equation of state.

At temperatures that are considerably in excess of the critical temperature of a fluid, and also at very low pressures, the vapour of the fluid tends to obey the equation;

$\frac{pv}{T} = constant = R.$ No gases in practice obey this law rigidly, but many gases tend towards it. An imaginary ideal gas which obey the law is called a perfect gas, and the equation , pv/T = R, is called the characteristic of state of a perfect gas. The constant R, is called the gas constant. The unit of R are N m/kg K or kJ/kg K. Each perfect gas has a different gas constant.

The characteristic equation is usually written;

$$pv = RT \ldots\ldots 1$$

or for m kg. occupying V m^3,

$$pV = mRT \ldots\ldots\ldots\ldots\ldots.. 2$$

Another form of the characteristic equation can be derived using the kilogram-mole as a unit. The kilogram-mole is defined as a quantity of gas equivalent to M kg. of the gas, where M is the molecular weight of the gas (e.g since the molecular weight of oxygen is 32, then 1 kg. mole of oxygen is equivalent 32 kg of oxygen). From the definition of kilogram-mole, for m kg of a gas we have,

$$m = nM \ldots\ldots\ldots\ldots\ldots\ldots 3$$

(where n is the number of moles)

Note: Since the standard of mass is the kg. kilogram-mole will be written simply as mole. Substituting for m from equation 3 in equation 2 gives;

$$pV = nMRT \text{ or } MR = \frac{pV}{nT}.$$

Now Avogadro's hypothesis state that the volume of 1 mole of any gas is the same as the volume of 1 mole of any other gas, when the gases are at the same temperature and pressure. Therefore V/n is the same for all gases at the same value of p and T. That is, the quantity pV/nT is a constant for all gases. This constant is called the universal gas constant and is given the symbol, R_o.

i.e $$MR = R_o = \frac{pV}{nT} \text{ or } pV = nR_oT \ldots\ldots\ldots\ldots\ldots\ldots 4$$

or since $MR = R_o$ then, $R = \frac{R_o}{M} \ldots\ldots\ldots\ldots\ldots.. 5$

Experiment has shown that the volume of one mole of any perfect gas at 1 bar and 0^0C is approximately 22.71 m^3. Therefore from equation 4,

$$Ro = \frac{pV}{nT} = \frac{1 \times 10^5 \times 22.71}{1 \times 273.15} = 8314.3 \text{ Nm/mole K.}$$

From equation 5 the gas constant for any gas can be found when the molecular weight is known, e,g for oxygen of molecular weight 32, the gas constant ,

$$R = \frac{R_o}{M} = \frac{8314}{32} = 259.8 \text{ N m/ kg K.}$$

Ex. 1. A vessel of volume 0.2 m^3 contain nitrogen at 1.013 bar and 15^0C. If 0.2 kg of nitrogen is now pumped into the vessel, calculate the new pressure when the vessel has returned to its initial temperature. The molecular weight of nitrogen is 28, and it may be assumed to be a perfect gas.

Solution: From equation 5,

Gas constant R = R_o / M = 8314/ 28 = 296.9 Nm/kg K,

From equation 2, for the initial conditions,

$$p_1V_1 = m_1RT_1.$$

$$\therefore m_1 = p1V_1 / RT_1 = \frac{1.013 \times 10^5 \times 0.2}{296.9 \times 288} = 0.237 \text{ kg. Where } T_1 = 15 + 273 = 288 \text{ K.}$$

0.2 kg of nitrogen are added hence, m2 = 0.2 + 0.237 = 0.437 kg. Then from equation 2, for the final conditions,

$$P_2V_2 = m_2RT_2.$$

But $V_2 = V_1$ and $T_2 = T_1$,

$$\therefore p_2 = \frac{m_2RT_2}{V_2} = \frac{0.437 \times 296.9 \times 288}{0.2} = \text{ i.e } p_2 = \frac{0.437 \times 296.9 \times 288}{10^5 \times 0.2} = 1.87 \text{ bar.}$$

Ex.2.

0.01 kg of a certain perfect gas occupies a volume of 0.003 m3 at a pressure of 7 bar and a temperature of 131^0C. Calculate the molecular weight of the gas. When the gas is allowed to expand until the pressure is 1 bar the final volume is 0.02 m3. Calculate the final temperature.

Solution: From equation 2,

$$p_1V_1 = mRT_1,$$

$$\therefore \quad R = \frac{p_1V_1}{mT_1} = \frac{7 \times 10^5 \times 0.003}{0.01 \times 404} = 520 \text{ N m/kg K. Where T1} = 131 + 273 = 404 \text{ K.}$$

Then from equation 5,

$$R = \frac{R_o}{M}, \quad \therefore M = \frac{R_o}{R} = \frac{8314}{520} = 16.$$ i.e Molecular weight = 16. From equation 2,

$$p_2V_2 = mRT2,$$

$$\therefore T_2 = \frac{p_2V_2}{mR} = \frac{1 \times 10^5 \times 0.02}{0.01 \times 520} = 384.5 \text{ K}$$

i.e, Final temperature = 384.5 – 273 = 111.5^0C.